### 10.1 Introduction to Probability

Write your questions here!


Probability is a measure of how likely an event will happen.

## Probability Scale



The smallest value a probability can be is $\qquad$ . The happens when the event will never happen.

The largest value a probability can be is $\qquad$ . This happens when the event will always happen.

Describe the likelihood of an event as impossible, unlikely, equally likely, likely or certain.

1) Our school closes today because of snow.

P(No School) $\qquad$
2) You get a "head" when flipping a coin.

P(Heads) $\qquad$
3) Roll \# less than 10 on 6 -sided number cube

P(\# < 10) $\qquad$
4) Your car will start tomorrow morning.

P(start) $\qquad$

Important definitions:

Event: $\qquad$

Outcome: $\qquad$

Likelihood: $\qquad$

Sample Space: $\qquad$

## Theoretical vs Experimentall Probability



Theoretical probability describes how
likely it is that an event will happen based on all the possible outcomes. It is what SHOULD happen and uses the ratio:
$\mathrm{P}($ event $)=\frac{\# \text { ways event could occur }}{\text { Number of possible outcomes }}$

Experimental probability is the probability based on experimental data that are found by repeating the experiment several times. It is what ACTUALLY happened and uses the ratio:
$\mathrm{P}($ event $)=\frac{\# \text { times event actually occurred }}{\text { Number of possible outcomes }}$

Roll a 6 -sided number cube 140 times.
How many times should you get a 4 ?

Roll a number cube 140 times. 28 landed on a 4.

Suppose Mr. Brust was nice enough to give you a pack of M\&Ms and you count and record how many of each color was in the bag.

M\&M Distribution

| Red | Orange | Yellow | Blue | Green | Brown | Purple |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 | 10 | 5 | 7 | 12 | 6 | 0 |

What is the experimental probability that you select a red M\&M? $\qquad$

P(Yellow) $\qquad$ $\mathrm{P}($ Green or Brown) $\qquad$ P (Purple) $\qquad$

P(Not Orange) $\qquad$ P (Blue or Red or Green) $\qquad$

## SUMMARY:



Describe the likelihood of an event as impossible, unlikely, equally likely, likely or certain.
Your Fortnite team wins $\frac{3}{4}$ of the time. $\qquad$
There is a $0 \%$ chance that you will grow 10 more feet. $\qquad$
The probability that the sun rises tomorrow is 1 . $\qquad$
It rains on $\frac{1}{5}$ of the days in July. $\qquad$
There is a $5 \%$ chance of winning a contest. $\qquad$
Picking a number less than 15 from a jar with papers labeled from 1 to 12 . $\qquad$
Picking an odd number from a jar with papers labeled from 1 to 12 . $\qquad$

Use the spinners to fill in the table below. Express each probability as a fraction in simplest form.

Spinner 1



| Probability | Spinner $\mathbf{~}$ | Spinner 2 | Spinner 3 |
| :---: | :---: | :---: | :---: |
| P(Yellow) |  |  |  |
| P(Blue) |  |  |  |
| P(Red) |  |  |  |

The letters of the word GOOSE are put into a bag. You randomly draw a letter from the bag. Find the probabilities as FRACTIONS.

$$
\begin{array}{lll}
P(\text { letter } S)= & P(\text { letter } G)= & P(\text { Letter } K)= \\
P(\text { vowel })= & P(\text { not a vowel })= & P(\text { letter } O)=
\end{array}
$$

Find each theoretical probability as a FRACTION in SIMPLEST FORM, if you roll a standard number cube.
$\qquad$ $P(2)=$ $\qquad$ $P(1$ or 2$)=$ $\qquad$
$P($ not a 4$)=$ $\qquad$
$P($ even number $)=$ $\qquad$ $P(7)=$ $\qquad$
Suppose a number cube is rolled 120 times. About how many times should each event occur?

Find the experimental probability of each event based off of the rolls of a number cube recorded in the table.

| \# on number cube | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \# of rolls | 16 | 20 | 13 | 17 | 19 | 15 |

$P(1)=$ $\qquad$
$P(\operatorname{Not} 2)=$ $\qquad$
$P($ Even $)=$ $\qquad$
$P(6)=$ $\qquad$

A bag of marbles contains: 1 green, 2 blue, 1 yellow, 3 purple and 3 red. Write each answer as a DECIMAL.
$P($ blue $)=$ $\qquad$
$P($ not red $)=$ $\qquad$ $P($ green $)=$ $\qquad$
$P($ not blue $)=$ $\qquad$
$\mathrm{P}($ purple $)=$ $\qquad$
$P($ blue or red $)=$ $\qquad$

1. Sully draws a pen at random from a bag of pens. He records its color and replaces it. His results are shown in the table below.

| Pens | Blue | Red | Black |
| :---: | :---: | :---: | :---: |
| Frequency | 29 | 19 | 27 |

## Find:



$$
\mathrm{P}(\text { Not red })=\frac{}{(\text { as a decimal })}
$$

$\mathrm{P}($ Black or Blue $)=\frac{}{(\text { as a fraction) }}$
$\mathrm{P}($ Green $)=\frac{}{\text { (as a decimal) }}$
2. Suppose a number cube is rolled 80 times. About how many times should each event occur?
a. You roll a 5 or a 6.
b. You roll a number greater than 1.
3. Your friend was absent and watched Mr. Sullivan's video on Probability. When they finished, they told you that the probability they would pass the Mastery Check was 2.6. How well do you think your friend will do? Explain.

## EXIT TICKET -

Suppose Sully and Bean went to the store and bought a large bag of M\&Ms. The following table represents the number and color of the M\&Ms.

M\&Ms Distribution

|  | Red <br> 22 | Orange <br> 18 | Yellow <br> 20 | Blue <br> 12 | Green <br> 10 | Brown <br> 30 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Fraction |  |  |  |  |  |  |
| Decimal |  |  |  |  |  |  |

a. Fill out the probabilities in the table.
b. What is the sum of the fraction probabilities? Show your work.
c. What is the sum of the decimal probabilities? Show your work.


TREE DIAGRAMS
SAMPLE SPACE OUTCOMES

Mr. Brust flips a coin twice. What is the probability he gets tails twice?

COMPOUND Probability is $\qquad$

We can find compound probabilities by multiplying the probabilities of each event.

A container holds 4 red pens, 3 purple pens, and 2 blue pens.

Use the above information to answer the following WITH replacement.

1) Find $P$ (red pen, purple pen)
2) Find $P$ (blue, red)


Use the above information to answer the following WITHOUT replacement.
4) Find $P$ (red pen, purple pen)
5) Find $P$ (blue, blue)


## Independent? Dependent?

When we have two events, we have to see if the outcome of the one event can change the probability of the other event.
$\qquad$ events have probabilities that never change.

If the probabilities of the two events can change, the events are $\qquad$ _.

You randomly choose a marble from a jar. You replace the marble (put it back in the jar) and randomly choose another marble.

You randomly draw a card from a deck of cards. You keep the card and then randomly draw another card.

INDEPENDENT or DEPENDENT

Suppose Mr. Brust was nice enough to give you a pack of M\&Ms and you count and record how many of each color was in the bag.

M\&M Distribution

| Red | Yellow | Blue | Green | Brown |
| :---: | :---: | :---: | :---: | :---: |
| 8 | 5 | 7 | 4 | 2 |

You select an M\&M and eat it. Then, you select another M\&M and eat that, too. (You must be hangry!) Find the following probabilities.

P(Red, then Blue) $\qquad$ P(Green, Green) $\qquad$

## Counting with Tree Diagrams

Mr. Brust is living large on the beaches of Sicily during summer. He only uses 3 different shirts (blue, green and red), 2 pair of shorts (black and white) and 2 type of shoes (flip flops, crocs). How many total outfits can Brust make?



Now, summarize your notes
here!

Tell whether the events are INDEPENDENT or DEPENDENT.

1. You spin a spinner. Then you flip a coin.
2. You randomly choose 1 of 10 marbles. Then you randomly choose one of the REMAINING 9 marbles.
3. You toss a coin twice.

INDEPENDENT or DEPENDENT
4. You spin a penny and a nickel on a table. The penny lands on heads and the nickel lands on tails.
5. A container has 7 green buttons, 3 yellow buttons and 4 blue buttons. You reach in and randomly draw out a blue button. You KEEP the blue button and reach in again to draw out a second blue button.

INDEPENDENT or DEPENDENT

A container holds $\mathbf{3}$ red pens, $\mathbf{6}$ black pens, $\mathbf{4}$ purple pens, and $\mathbf{2}$ blue pens.
Use the above information to answer the following WITH replacement.
6. Find P (red pen, purple pen)
7. Find P (blue and then red)
8. Find P (black, black)

Use the above information to answer the following WITHOUT replacement.
9. Find P (red pen, purple pen)
10. Find P (blue and then red)
11. Find P (black, black)

Find each probability.

Each item is NOT REPLACED.
12. A box contains 6 red and 5 blue pencils.

Choose a red one, keep it, and choose another red one.

Find each probability. Each item IS REPLACED.
13. A box contains 6 red and 5 blue pencils Choose a red one and choose another red one.

The colors of M\&Ms in a large bag are given in the table.
M\&M Distribution

| Med |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Yellow | Blue | Green | Brown |  |
| 2 | 5 | 7 | 6 | 10 |

14. You draw 2 M\&M's without replacement. Find $P(2$ Greens) as a fraction and as a decimal.
15. You draw 2 M\&M's with replacement. Find $\mathrm{P}(2$ Greens) as a fraction and as a decimal.

Use the spinner shown below for questions 16-19.
16. Find P (spin Blue).
17. Find $P($ spin Red $)$.

18. Find P (spin Green and then Blue)
19. What is the sample space of 2 spins? Hint: There are 9 outcomes!

Bob has 5 books about cats, 6 books about dogs, and 2 books about fish. Use this to answer 22-24.
20. How many different ways can Bob select one book on cats, one on dogs, and one of fish?
21. Bob randomly grabs one of his books. Find $\mathrm{P}($ fish book).
22. Bob randomly grabs two of his books (without replacement). Find P (two cat books).

## Find each probability.

## Each item is NOT REPLACED.

1. A box contains 3 white and 5 blue marbles.

Choose a white one, keep it, and choose a blue one.

## Each item IS REPLACED.

2. A bag contains 10 blue and 15 red M\&Ms Choose a red one and choose and other red one.
3. Bob can order a Pepsi or Fanta in a Small, Medium or Large. How many different sodas can Bob possibly order? Draw a tree diagram and list all possible outcomes.

## EXIT TICKET -

The colors of $\mathrm{M} \& \mathrm{Ms}$ in a large bag are distributed according to the probabilities shown in the table:

| Color | Brown | Red | Yellow | Green | Orange | Blue |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Probability | 0.25 | 0.25 | 0.20 | 0.10 | 0.10 | $?$ |

Fill in the table by finding P(blue). (HINT: ALL of The probabilities add to what number?)

You draw 2 M\&M's, with replacement. Find P(red and then orange).

## MATH 7



COIN: HEADS, TAILS


NUMBER CUBE:

## Simulations

Suppose having a baby boy is equally likely as having a baby girl. If Franky has three children, how likely is it that all of the children are the same gender?

If we found 200 parents who all had 3 children, about how many would have children who are all the same gender?

Nia makes 5 out of every 6 free throws in basketball. How likely is it that Nia misses two shots at the end of a game?

If Nia has to make the last two shots 20 times, about how many games will she win?

## Canadian Spinners

Suppose Paul spins the spinner 400 times and gets 142 A's. Is this reasonable?


## More Tree Diagrams:

Sully loves a good breakfast! He always has either a bagel, or a muffin. To drink, he chooses between milk, juice and coffee. Estimate about how many times Sully would have a bagel with milk if he selects randomly for 365 days.

## SUMMARY:



Janice has a teacher that only collects the homework $50 \%$ of the time. Janice gets $\mathbf{3}$ homework assignments every week. Janice wonders how many homework assignments his teacher will collect this week?

1. List all possible outcomes like "collect, no collect, collect". You can use a tree diagram to represent them!
2. What is the theoretical probability that Janice's teacher will collect all 3 assignments?
3. Find P (collect, collect, no collect).

Janice decides to simulate her teacher's homework collection by tossing a coin three times. Heads means her teacher collects the homework, tails means her teacher does not collect the homework. She records her result below.

$$
\begin{aligned}
& \text { Simulation } 1=\text { H H T } \\
& \text { Simulation } 2=\text { T H T } \\
& \text { Simulation } 3=\text { T T T }
\end{aligned}
$$

4. According to simulation \#1, how many times would Janice's teacher collect her homework?
5. According to simulation \#2, how many times would Janice's teacher collect her homework?

Janice runs her simulation 500 times. 64 times Janice's teacher would collect the homework all three days.
6. Use this simulation to predict how many weeks his teacher would collect the homework all three days in 28 weeks.
7. Set up a simulation using a six-sided number cube for Janice's teacher homework collection. Explain in detail how your simulation would work!

## Probability

8. $45 \%$ of people that go to the movies buy popcorn. If there are 160 people at the new Star Wars movie, how many will have popcorn?
9. The Bengals win 9 out of 16 games played. How many games will they win out of 80 games?
10. You sit at the food court and count red headed people. You see 2 red heads out of 50 people. If there are 400 people at the mall, how many would be red headed?

Josh Allen thinks that you are equally likely to have brown, blue, green or hazel colored eyes. He creates a simulation using a deck of cards. Each suite represents an eye color. Bob draws a card and then replaces it. Use his results to answer the following.
11. Find $\mathrm{P}($ Blue Eyes $)$.

| Hearts (Brown Eyes) | HY II |
| :---: | :---: |
| Diamonds (Blue Eyes) | H 1111 |
| Spades (Green Eyes) | H H H I |
| Clubs (Hazel Eyes) | \|1] |

12. Out of 350 people, how many would you expect to have blue eyes based on your simulation?
13. Find $\mathrm{P}($ Green Eyes $)$.
14. Out of 350 people, how many would you expect to have green eyes based on your simulation?
15. Turns out, only $17 \%$ of people have blue eyes. Out of 350 people, how many would you expect to have blue eyes based on the theoretical probability?
16. $10 \%$ of people are left handed. There 240 people at school today. How many are left handed?
17. Two out of every 6 students bring Mr. Brust a gift for teacher appreciation week. Explain how to use a number cube (dice) to simulate the number of students out of 80 that bring him a gift for teacher appreciation week.

Bob goes to Las Vegas. He watches a slot machine all day and notices that the slot machine only hits Jackpot 4 times when played 315 times. Use this experimental probability to answer the following.
3. Bob plays the slots 40 times. How many jackpots should he hit?
4. The casino has hundreds of people playing. A slot room on a normal day could have 45,000 plays. How many jackpots would the room have the entire day?
5. Big casinos have multiple slot machine rooms. A big casino may have 820,000 plays. How many jackpots would that casino have?

One Casino claims their slots hit Jackpot $\mathbf{2 \%}$ of the time. Use this theoretical probability to answer...
6. Bob plays the slots 40 times. How many jackpots should he hit?
7. The casino has hundreds of people playing. A slot room on a normal day could have 45,000 plays. How many
Jackpots would the room have?
8. Big casinos have multiple slot machine rooms. A big casino may have 820,000 plays. How many Jackpots would the casino have?

## EXIT TICKET -

Bob wants to create a simulation using a deck of cards. Since a deck of cards has 52 cards, he removes two cards so that there are only 50 cards. Then, because $\frac{1}{50}$ cards $=2 \%$, he says that the Ace of Hearts will represent "Jackpot".

Bob runs his simulation 500 times and gets 12 Ace of Hearts. Use this simulation to answer the following.
9. Bob plays the slots 40 times. How many jackpots should he hit?
10. The casino has hundreds of people playing. A slot room on a normal could have 45,000 plays. How many Jackpots would the room have?
$\qquad$
Describe the likelihood of an event as impossible, unlikely, equally likely, likely or certain.

1. Your football team wins $\frac{1}{5}$ of the time.
2. There is a $90 \%$ chance that you pass this test.
3. The probability that the sun rises in the west tomorrow is 0 .
4. Picking an even number from a jar with papers labeled from 1 to 5 .
$\qquad$
papers labed from 1 to 5

Find each theoretical probability as a FRACTION in SIMPLEST FORM, if you roll a standard number cube.
5. $P(\operatorname{not} 2)=$ $\qquad$
6. $P(5$ or 6$)=$ $\qquad$
7. $P(8)=$ $\qquad$

Suppose a number cube is rolled 220 times. About how many times should each event occur?
8. A 4 is rolled. 9. An even number is rolled.

After the number cube was rolled, the following outcomes were recorded:
Find the experimental probability of each event based off of the flipping of a coin recorded in the table. Write your answers as fractions in lowest terms.

|  | \# of <br> flips |
| :---: | :---: |
| Heads | 16 |
| Tails | 18 |

10. $\quad$ P (Tails $)=$ $\qquad$ 11. $\mathrm{P}($ Two Tails if flipped twice $)=$ $\qquad$


A bag of marbles contains: 12 green, 1 blue, 2 yellow, and 1 purple. Find each probability as a DECIMAL.
12. $\mathrm{P}($ green $)=$ $\qquad$
13. $\mathrm{P}($ green or blue $)=$ $\qquad$

Tell whether the events are INDEPENDENT or DEPENDENT.
14. You roll a number cube twice. You get a 4 an a 1 .

INDEPENDENT or DEPENDENT
15. You toss a coin. If it is heads, you toss it again. If it is tails, you quit.

INDEPENDENT or DEPENDENT

A bag holds 3 green, 2 blue, and 5 magenta pens. You select a pen randomly.
Use the above information to answer the following WITHOUT replacement. 16. Find $P$ (green, green) 17. 31. Find $P$ (magenta, green)

Use the above information to answer the following WITH replacement.
18. Find P (green, green)
19. 31. Find $P$ (magenta, green)
20. The Cleveland Browns win 9 out of 16 games played. How many games will they win out of 100 games?

In middle school, Brust would go to a school dance and "Brust a Move" (dance) 50\% of the time. The other $50 \%$ of the time he would stay home and read comic books. Suppose Brust's school had 3 dances one year.
21. Make a tree diagram to show all of the possible outcomes for going to the 3 dances. Then, list each outcome lie "DANCE, READ COMIC, DANCE"

Tree Diagram:

Outcomes:
41. What is the theoretical probability that Brust goes to all three dances?

Sully wants to know if he will see Mr. Brust at the dances. He simulates the outcome of the three dances using a coin. A heads represents "Brust a Move!" and a tails represents "Reads Comics". Sully records the results here:

| Simulation \#1: | HTH | Simulation \#2: | THT | Simulation \#3: | HHH |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Simulation \#4: | TTH | Simulation \#5: | THH | Simulation \#6: | THH |  |
| Simulation \#7: | HTT | Simulation \#8: | TTT | Simulation \#9: | HTT | Simulation \#10: THT |

43. According to simulation \#8, how many dances did Brust attend?
44. According to the simulations, what is the experimental probability that Brust attends all 3 dances?
